BACKPAPER EXAMINATION
M. MATH II YEAR
FOURIER ANALYSIS
I SEMSTER, 2014-15

The seven questions below carry a total of 110 marks. Answer as many questions as you can.
The maximum you can score is 100. Time limit is 3 hours.

1. Let $f = I_{(-1,1)}$ considered as an element of $L^1(\mathbb{R})$ and $g = f * f$. Show that the Hilbert transform Hg is given by $Hg(x) = c\int\limits_0^\infty \sin(tx)(\frac{\sin t}{t})^2 dt$ where c is an absolute constant. [20]

2. Let M be a closed subspace of $L^1(\mathbb{R})$ considered as a ring under convolution as multiplication. Show that M is an ideal if and only if $f_a \in M$ whenever $f \in M$ and $a \in \mathbb{R}$. [20]

3. If $f, g \in L^1(\mathbb{R}), \|g\|_1 < 1$ and $f = f * g$ show that $f = 0$ a.e. [10]

4. Compute explicitly the Fourier transforms of $\frac{\sin x}{x}$ and $\left(\frac{\sin x}{x}\right)^2$. [20]

5. Prove that if $f \in L^1(\mathbb{R})$ and $f, \hat{f}$ both have compact support then $f = 0$ a.e. [15]

6. Let $X = L^2(\mathbb{R})$ and M be the closed subspace of X generated by $\{I_{A_n} : n \geq 1\}$ where $\{A_n : n \geq 1\}$ is a partition of $\mathbb{R}$ by Lebesgue measurable sets. Write a formula for the projection of an element f of X on M and jusitfy your formula. [10]

7. Prove that $\{f \in L^1(\mathbb{R}) : \hat{f}$ has compact support$\}$ is dense in $L^2(\mathbb{R})$. [15]